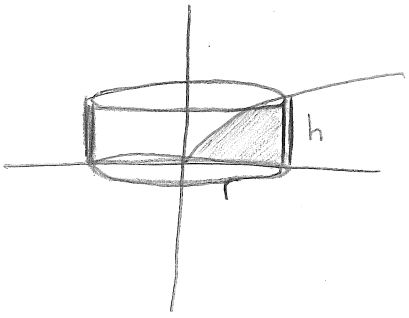
AP Calculus AB
Shells Method Notes

Name Key

With this method, you draw a line segment parallel to the axis of rotation. The solid formed is a cylinder.

Surface area of a cylinder = $2\pi rh$

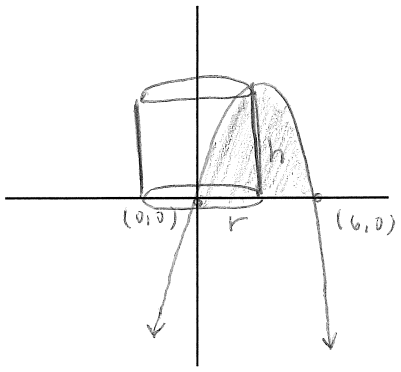


about y-axis:
- segment drawn parallel to the axis of rotation
  ↳ height (area between curves)
  radius - distance from the axis of rotation

Volume = $2\pi\int_a^b x[f(x)-g(x)]dx$ where x is the radius and $[f(x)-g(x)]$ is the height

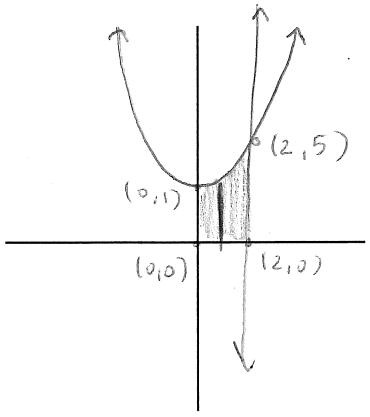
Examples:
Find the volume.

1. $y = 6x - x^2$, x-axis about the y-axis



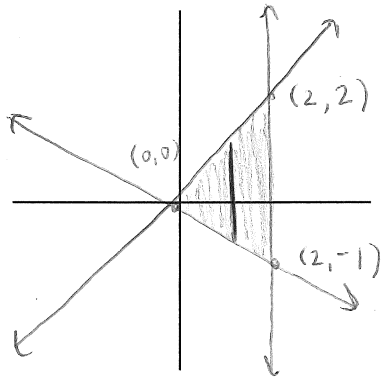
$2\pi\int_0^6 (x)(6x-x^2)\,dx$

2. $y = x^2 + 1$, x=2, x-axis, y-axis about the y-axis



$2\pi\int_0^2 (x)(x^2+1)\,dx$

3. $y = x$, x = 2, $y = -\frac{x}{2}$ about the y-axis



$$2\pi \int_0^2 (x)\left(x + \frac{x}{2}\right) dx$$

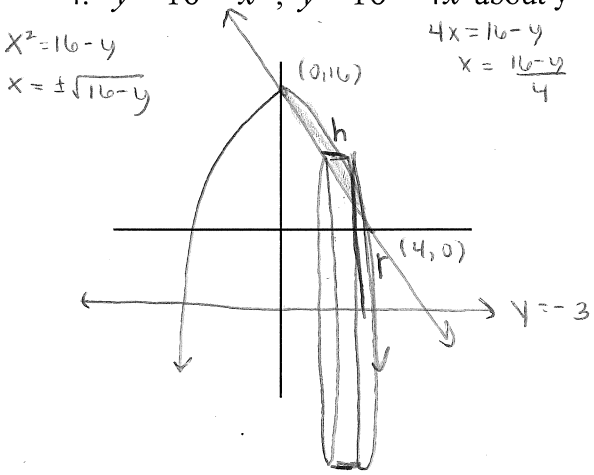
4. $y = 16 - x^2$, $y = 16 - 4x$ about y = -3

$x^2 = 16 - y$

$x = \pm\sqrt{16-y}$

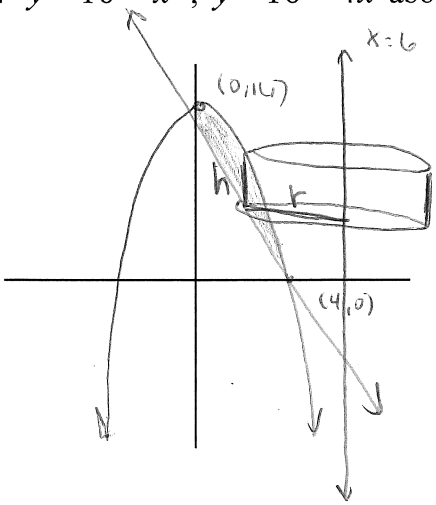
$4x = 16 - y$

$x = \frac{16-y}{4}$



$$2\pi \int_0^{16} (y+3)\left(\sqrt{16-y} - \frac{16-y}{4}\right) dy$$

5. $y = 16 - x^2$, $y = 16 - 4x$ about x = 6



$$2\pi \int_0^4 (6-x)\left(16 - x^2 - (16 - 4x)\right) dx$$

$$2\pi \int_0^4 (6-x)(-x^2 - 4x) dx$$